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ABSTRACT
When using the conventional single-domain boundary element method in an acoustic interior problem, even a slight change of the cavity geometry affects the whole system matrix. If the design variables are related only to a small portion of the whole cavity, the geometric difference of the cavity shape that appears during the intermediate iteration process before reaching the optimal shape is usually small compared to the original cavity shape. The iterative process in this sensitivity analysis problem can be somewhat easily handled by implementing the multi-domain BEM (MBEM) technique. When using the sub-domain modularization scheme of the MBEM, the interior domain of cavity can be divided into several sub-domains under two categories: the sub-domain without change of geometry and the other sub-domains that will evolve its geometry along the change of preset design variables. It is thought that the computational efficiency can be improved by using this technique. A demonstration example is given by a two-dimensional automotive interior cavity.

KEYWORDS: BEM, Multi-domain BEM, Module, Sensitivity analysis, Interior problem

INTRODUCTION
Numerical methods for optimum design are conceptually different from the analytical
methods. The numerical optimization process is based on the iterative design process, i.e., the optimum design is determined by some number of iterative changes from the initial design. For efficient iterations, the effective search directions and step sizes in the space of design variables should be determined by using the shape design sensitivities (SDS). Ding [1] and Kwak [2] presented an excellent review on the shape optimal design and sensitivity analysis. The acoustic shape design sensitivity analysis (ASDSA) is the process of computing the rate of change of acoustic response as an object function or a constraint value with respect to changes of the design parameters. Either the finite element method (FEM) [3] or the boundary element method (BEM) can be used as a numerical method to calculate the sensitivities. For general interior problems, the FEM is more popular than the BEM. However, the BEM has advantages compared with the FEM in terms of accuracy on the boundary and the ease of modification of mesh during iterations.

As a numerical method, the multi-domain BEM (MBEM) is now utilized to calculate the SDS effectively and to carry out the fast and effective iteration process for shape optimization of the practical problems. The MBEM can be regarded as the general form of BEM, in which the interior cavity is divided into several sub-domains connected by the hypothetical interface planes. The boundary surfaces, which include the imaginary interfaces, of each sub-domain do not fold back onto themselves and constitute a complete closed loop or cavity. By using the single-domain boundary integral equations of each sub-domain and the continuity conditions (pressure and normal velocity) on the imaginary interfaces, the acoustic pressures and normal velocities on all surfaces including imaginary interfaces can be obtained and, then, every sub-domain can be treated as the independent single-domain.

When using the conventional single-domain boundary element method (SBEM), slight changes of the cavity geometry affects the whole system matrix. Due to this fact, the remeshing process would be quite ineffective. If the design variables occupy the small portions of the whole cavity, the geometric differences between the original cavity and the intermediate ones, including optimum one, during the iteration process are small. When using the sub-domain modularization scheme [4] of MBEM, the interior domain of cavity can be divided into several sub-domains under two categories: the sub-domain having the same geometry, which is not affected by the change of design variables and the other sub-domains having different geometries, that are affected by the change of design variables. This feature could be utilized to get beneficial result in improving the computational efficiency when combined with the use of the MBEM.

In this paper, acoustic design sensitivity formulations based on the MBEM are presented by differentiating the discretized MBIE. Each sub-domain is divided by the rule of the sub-domain modularization scheme. First, the sensitivities of the surface variables are calculated
by solving the combined matrix equation with the finite difference scheme. Then, the sensitivities of the field pressures are obtained by using the acquired surface variables and sensitivities. The computational efficiency could be improved by using this technique and this fact was demonstrated through an example of a two-dimensional automotive interior cavity.

**DESIGN SENSITIVITY ANALYSIS USING THE FINITE DIFFERENCE SCHEME FOR MBEM**

**Design sensitivities of the surface variables**

The linear system matrix equation of MBEM for all types of boundary conditions has already formulated in [4]. Fig. 1 shows a schematic diagram for the design sensitivity analysis using the MBEM. $S_{1N}$ and $S_{1I}$ denote the surface with Neumann boundary condition and the imaginary interface, respectively, of the first sub-domain. In this case, $S_{1I}$ and $S_{2I}$ are identical to each other, but their normal directions are opposite. If the Neumann boundary condition is only imposed on all surfaces, the system matrix equation can be rewritten as

$$
\begin{bmatrix}
-A_1^{NN} & -A_1^{NI} & B_1^{NI} & 0 \\
-A_1^{IN} & -A_1^{II} & B_1^{II} & 0 \\
0 & -A_2^{NI} & -B_2^{NI} & -A_2^{NN} \\
0 & -A_2^{II} & -B_2^{II} & -A_2^{IN}
\end{bmatrix}
\begin{bmatrix}
\phi_1^N \\
\phi_1^I \\
\phi_2^N \\
\phi_2^I
\end{bmatrix}
=
\begin{bmatrix}
-B_1^{NN} & \phi_1^{N} \\
-B_1^{IN} & \phi_1^{I} \\
-B_2^{NN} & \phi_2^{N} \\
-B_2^{IN} & \phi_2^{I}
\end{bmatrix},
$$

where $A_{R(i,j)}^{QQ} = A_{R(i,j)}^{QQ} - c_{R(i)} \delta_{(i,j)}$, $\delta_{(i,j)}$ is the Kronecker delta ( $\delta_{(i,j)} = 0$ for $i \neq j$, $\delta_{(i,j)} = 1$ for $i = j$) and $c_{R(i)}$ is the solid angle on $i$th node of $S_{RQ}$ in $\Omega_R$. $A_{RQ}^{PO}$ and $B_{RQ}^{PO}$ represent dipole and monopole matrices when the field point $r$ is located on $S_{RP}$ and the surface point $r_0$ on $S_{RQ}$, respectively. $\phi_1^Q$ and $\phi_2^Q$ are the velocity potentials and the normal derivatives of the velocity potentials of $S_{RQ}$, respectively ($R = 1,2,P = N,I,$ and $Q = N,I$). For convenience, Eq. (1) can be rewritten by partitioning as

$$
\begin{bmatrix}
H_1 & H'_1 & -G'_1 \\
0 & H'_2 & G'_2 & H_2
\end{bmatrix}
\begin{bmatrix}
\phi_1^N \\
\phi_1^I \\
\phi_2^N \\
\phi_2^I
\end{bmatrix}
=
\begin{bmatrix}
G_1 \cdot \phi_1^{N} \\
G_2 \cdot \phi_2^{N}
\end{bmatrix},
$$

where

$$
H_1 = \begin{bmatrix}
-A_1^{NN} \\
-A_1^{IN}
\end{bmatrix}, \quad H'_1 = \begin{bmatrix}
-A_1^{NI} \\
-A_1^{II}
\end{bmatrix}, \quad G'_1 = \begin{bmatrix}
-B_1^{NI} \\
-B_1^{II}
\end{bmatrix}, \quad G_1 = \begin{bmatrix}
-B_1^{NN} \\
-B_1^{IN}
\end{bmatrix},
$$

(3a-d)
The design variable \( a \) is included in the second sub-domain \( \Omega_2 \) and the design variable \( b \) is included in the first sub-domain \( \Omega_1 \) (See Fig. 1). In order to obtain the design sensitivities of the surface variables with respect to design variable \( a \), the partial derivative with respect to \( a \) should be applied on Eq. (2). When using the MBEM with the sub-domain modularization scheme, \( \mathbf{H}_R, \mathbf{H}_R^I, \mathbf{G}_R, \) and \( \mathbf{G}_R^I \) \((R=1,2)\) are functions of only the geometry of the \( R^{th} \) sub-domain \( \Omega_R \) and frequency. If the design variable \( a \) is relevant to the shape of the cavity and is included in \( \Omega_2 \), then \( \mathbf{H}_1, \mathbf{H}_1^I, \mathbf{G}_1, \) and \( \mathbf{G}_1^I \) are independent from the shape design variable \( a \). Thus, the derivatives of \( \mathbf{H}_1, \mathbf{H}_1^I, \mathbf{G}_1, \) and \( \mathbf{G}_1^I \) with respect to \( a \) turn out to be zeros such that

\[
\frac{\partial \mathbf{H}_1}{\partial a} = \frac{\partial \mathbf{H}_1^I}{\partial a} = \frac{\partial \mathbf{G}_1^I}{\partial a} = \frac{\partial \mathbf{G}_1}{\partial a} = 0.
\] (4)

This means that there is no need to re-mesh the first sub-domain during the sensitivity analysis with respect to the shape design variable \( a \). However, \( \mathbf{H}_2, \mathbf{H}_2^I, \mathbf{G}_2, \) and \( \mathbf{G}_2^I \) are function of the shape design variable \( a \). Therefore, the derivatives of these matrices with respect to \( a \) do not vanish anymore. They can be obtained using the FD scheme as

\[
\frac{\partial \mathbf{H}_2}{\partial a} = \frac{[\mathbf{H}_2]_{j+1} - [\mathbf{H}_2]_{j}}{\Delta a}, \quad \frac{\partial \mathbf{G}_2^I}{\partial a} = \frac{[\mathbf{G}_2^I]_{j+1} - [\mathbf{G}_2^I]_{j}}{\Delta a}
\]

\[
\frac{\partial \mathbf{H}_2}{\partial a} = \frac{[\mathbf{H}_2]_{j+1} - [\mathbf{H}_2]_{j}}{\Delta a}, \quad \frac{\partial \mathbf{G}_2}{\partial a} = \frac{[\mathbf{G}_2]_{j+1} - [\mathbf{G}_2]_{j}}{\Delta a}.
\] (5a–d)

Here, \([\mathbf{H}_2]_{j}, [\mathbf{H}_2^I]_{j}, [\mathbf{G}_2]_{j}, \) and \([\mathbf{G}_2^I]_{j}\) are the system matrices calculated from the original MBEM model of the second sub-domain module and \([\mathbf{H}_2]_{j+1}, [\mathbf{H}_2^I]_{j+1}, [\mathbf{G}_2]_{j+1}, \) and \([\mathbf{G}_2^I]_{j+1}\) are those from the re-meshed MBEM model of the second sub-domain module with \( \Delta a \). The final matrix equation can be derived as follows:

\[
\begin{bmatrix}
\mathbf{H}_1 & \mathbf{H}_1^I & -\mathbf{G}_1^I \\
0 & \mathbf{H}_2 & \mathbf{G}_2^I \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \mathbf{\phi}^N}{\partial a} \\
\frac{\partial \mathbf{\phi}^I}{\partial a} \\
\end{bmatrix}
= \begin{bmatrix}
\mathbf{G}_1 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\partial \mathbf{H}_2}{\partial a} & \frac{\partial \mathbf{G}_2}{\partial a} & \frac{\partial \mathbf{H}_2^I}{\partial a} & \frac{\partial \mathbf{G}_2^I}{\partial a} & \frac{\partial \mathbf{G}_2}{\partial a} \\
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \mathbf{\phi}^N}{\partial a} \\
\frac{\partial \mathbf{\phi}^I}{\partial a} \\
\end{bmatrix}.
\] (6)
Here, the system matrix of LHS of Eq. (6) is identical to that of Eq. (2). Thus, the additional calculation for obtaining the inverse matrix is not needed. \( \partial \phi_i^N / \partial a \) and \( \partial \phi_j^N / \partial a \) are given as the boundary condition in this study.

**Design sensitivities of the field variables**

When calculating the design sensitivities of the field variables, the sensitivities of the surface variables obtained above are used as given values. Each sub-domain can be treated as a completely closed single domain after all unknowns on the surfaces including the imaginary interface were obtained by solving Eq. (2). Therefore, the field pressure at a field point can be calculated by solving Eq. (7) derived from the sub-domain including the field point:

\[
\phi_F = A_F \phi_S - B_F \phi'_S .
\]  

Here, \( \phi_F \) is the velocity potential at the field point, \( \phi_S \) and \( \phi'_S \) are the velocity potential and its normal derivative on the boundary surfaces including the imaginary interface, respectively. \( A_F \) and \( B_F \) are the transfer matrices between the dipole and monopole sources on the surfaces and the field points, respectively, and they are function of the geometry of the sub-domain, the frequency, and the location of field point. In this case, the location of field point is very important.

If the field point is located in the first sub-domain \( \Omega_1 \) and the design variable \( a \) is included in the second sub-domain \( \Omega_2 \), then the derivative of Eq.(7) with respect to \( a \) is given by

\[
\frac{\partial \phi_F}{\partial a} = A_F \frac{\partial \phi_S}{\partial a} + A_F \frac{\partial \phi'_S}{\partial a} - B_F \frac{\partial \phi'_S}{\partial a} - B_F \frac{\partial \phi'_S}{\partial a} \quad \text{in } \Omega_1 .
\]  

However, \( A_F \) and \( B_F \) of \( \Omega_1 \) are independent from the design variable \( a \) which is included in \( \Omega_2 \). Thus, \( \partial A_F / \partial a \) and \( \partial B_F / \partial a \) turn out to be zeros. Then, Eq.(8) can be rewritten as

\[
\frac{\partial \phi_F}{\partial a} = A_F \frac{\partial \phi_S}{\partial a} - B_F \frac{\partial \phi'_S}{\partial a} \quad \text{in } \Omega_1 .
\]  

Contrastingly, if the field point is located in the second sub-domain \( \Omega_2 \) and the design variable \( a \), also, is included in the second sub-domain, \( \Omega_2 \), then the derivative of Eq.(7) with respect to \( a \) can be written as

\[
\frac{\partial \phi_F}{\partial a} = A_F \frac{\partial \phi_S}{\partial a} + A_F \frac{\partial \phi'_S}{\partial a} - B_F \frac{\partial \phi'_S}{\partial a} - B_F \frac{\partial \phi'_S}{\partial a} \quad \text{in } \Omega_2 .
\]  

Additionally, \( A_F \) and \( B_F \) of \( \Omega_2 \) are not independent from the design variable \( a \) which
is included in $\Omega_2$ and is relevant to the shape of $\Omega_2$. Thus, $\frac{\partial A_F}{\partial a}$ and $\frac{\partial B_F}{\partial a}$ do not vanish and can be obtained by using the FD scheme.

**SIMULATIONS**

As an example of irregular domains, a simple two-dimensional automotive interior cavity is taken for optimally reducing the noise level at the driver’s ear position. Fig. 2(a) shows the SBEM model having 120 quadratic line elements and 240 nodes. The element length is $0.12m$. The simplified front dash panels are harmonically vibrated with the uniform velocity, $v_n = 0.01 m/s$ for all frequencies and the rigid boundary conditions are imposed on all other surfaces. It was intended to optimize the depths of the cavity between the rear seat and the tailgate $a$ for minimizing the mean-squared acoustic pressure at the driver’s ear position at a certain frequency. The driver’s ear position is represented by a region $C$, which consists of 10 field points. Fig. 2(b) illustrates the MBEM model having two sub-domains divided by the imaginary interface between the roof and the top of rear seat. The first sub-domain consists of 102 quadratic line elements and 204 nodes and the second sub-domain consists of 26 quadratic line elements and 52 nodes. In this case, the design variable $a$ is included in the second sub-domain and the field points (region $C$) are included in the first sub-domain. Thus, Eq.(9) was employed to calculate the field pressure sensitivities. During the iterations for the optimal design, the second sub-domain was the only target for modifications.

Fig. 3 compares the sound field of optimal shape to that of initial one at 105 Hz. The mean-square pressure in region $C$ was minimized and one can observe that the quiet region is adjusted from the front of the driver to the driver’s head position. It was found that the length of $a$ became longer than the initial ones at 105 Hz because the frequency was higher than the third longitudinal cavity mode (93.2 Hz). In this optimization process, the size of perturbation of the design variable $\Delta a$ was 0.0001 m. Until the optimum design ($a = 0.7251 m$) was found, the total numbers of iterations when using the SBEM or MBEM were identically as 89.

Table 1 shows a comparison of the computational times in using the SBEM and MBEM in the cases of the first iteration from the initial condition and during 89 iterations. When using the SBEM, each iteration process spends nearly the same computational time because the change of the design variable affects the whole system matrix. When using the MBEM, the first iteration process took approximately same time with that in using the SBEM. However, after the first iteration process, the required computation times of further iterations became much smaller because the system matrices (including the inverse matrix) use the first sub-domain without additional repetitive works and the re-meshing was given to the second sub-domain.
only. Consequently, the use of the MBEM for the shape optimal design is much more effective than that of the SBEM.

**CONCLUSIONS**

The acoustic design sensitivities of the surface variables and field variables with respect to the shape design variables were formulated by differentiating the MBIE with respect to the specific design variable. During the procedure in finding the design sensitivities, the process of mesh modification was needed for calculating the derivatives of the sub-system matrices. In this case, the location of field point was important and affected the efficiency of calculation. Consequently, when using the MBEM for the design sensitivity analysis, it is effective, in the viewpoint of the computational cost, that the field point would be located in the sub-domain, in which the shape design variables are not included. As an example for an irregular cavity, a simple two-dimensional automotive interior cavity was taken for optimally reducing the noise level at the driver’s ear position. It was found that the use of the MBEM for the shape optimal design is much more effective than that of the SBEM.

**REFERENCES**


Table 1. A comparison of the spent computational times for optimizing the design variable of two dimensional car cavity by using the SBEM and MBEM by using a PC (Alpha 21164 microprocessor 533 MHz). Two computation conditions are given: single calculation from the initial condition and 89 iterations.

<table>
<thead>
<tr>
<th></th>
<th>1st iteration</th>
<th>89th iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>SBEM</td>
<td>9.23s</td>
<td>836.62 s</td>
</tr>
<tr>
<td>MBEM</td>
<td>7.31s</td>
<td>209.71 s</td>
</tr>
</tbody>
</table>
Fig. 1. A schematic diagram for the design sensitivity analysis using the MBEM.

Fig. 2. Two-dimensional rigid wall model of an automotive interior cavity. Excitation will be given to the section A-B with a uniform velocity. (a) SBEM model, (b) MBEM model.

Fig. 3. Calculated mean-squared acoustic pressure distribution in the car cavity at 105 Hz. (a) Initial shape ($a=0.5\ m$), (b) optimal shape ($a=0.7251\ m$).